# Log-stable laws as asymptotic solutions to a fragmentation equation: Application to the distribution of droplets in a high Weber-number spray

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In this paper, it will be shown that "totally skewed to the left" log-stable distributions are suitable asymptotic solutions to a fragmentation equation. This result generalizes Kolmogorov's work on log-normal distribution for the drops' size number distribution of particles under pulverization. Indeed, Kolmogorov's discrete process is extended to a continuous time Markov process for the volume distribution instead of the number distribution. New hypotheses are then introduced which lead to log-stable distributions as asymptotic solutions of the fragmentation equation. Log-stable laws are then used to fit experimental probability distribution function (pdf) of Simmons and Hanratty measuring drop sizes in a horizontal annular gas-liquid flow at high Weber number [Int. J. Multiphase Flow **27**, 861 (2001)]. Log-stable pdf better fits to the experimental pdf than usual empirical spray pdf and especially, because of the heavy tail of the associated stable distribution, in the small drops part of the distribution.

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## I. INTRODUCTION

There is a real industrial incentive in developing soundly founded drop spray probability distribution function (pdf). Commonly used pdf's are Rosin-Rammler (akin to Weibull pdf), modified Rosin-Rammler (akin to log-Weibull pdf), or upper-limit log-normal pdf of Mugele and Evans [1]. But all of these, except the seldom used log-normal pdf, are built on empirical grounds. In this paper, a fragmentation equation is proposed which possesses asymptotically self-similar log-stable drop volume pdf solutions. This result may be used for modeling high Weber-number sprays.

Related works, on atomization, date back to Kolmogorov [2] who proposed in 1941 a discrete Markov process explaining why the log-normal distribution could be used as a number pdf for particles during pulverization. Oboukhov [3] applied this modeling to turbulence intermittencies, which was one of the roots of the "refined similarity hypothesis" developed in 1962 by Kolmogorov [4]. More recently, Novikov proposed to apply some ideas on turbulence intermittencies [5] back to turbulent spray modeling through the use of infinitely divisible pdf [6], of whom stable distributions form a small subset. However, infinitely divisible distributions are too generic to be easily handled or computed. In a related way, Zhou and Yu [7] proposed a multifractal analysis of the spray generated by an air-blast nozzle but without relating it explicitly to the spray pdf. Lastly, the work of Kolmogorov has been extended to continuous time and further analyzed by Gorokhovski and Saveliev [8]. Their continuous Markov process is very similar to Kostoglou's [9] fragmentation equation with a homogeneous kernel. Finally, let us pinpoint that log-stable distributions have already been used in turbulence intermittency modeling by Kida [10,11] and Schertzer and co-workers [12].

In the first part of this paper, we generalize Kolmogorov's work from *number* pdf to *volume* pdf using the formalism developed by Gorokhovski and Saveliev [8]. It is then shown that replacing Kolmogorov's hypothesis of finiteness of the first three moments by assuming that there exists a real number $\alpha$ such that moments of order $\alpha$ and above are infinite, and it leads asymptotically to a log-stable volume pdf. In the second part, this modeling is applied to the distribution of droplets in a high Weber number spray: a log-stable volume drops pdf is fitted to spray pdf obtained by Simmons and Hanratty [13] in a horizontal annular gas-liquid flow. Comparison is then made between these pdf's and the most widely used ones in atomization (log-Weibull, upper-limit Evans, and log-normal). The good performance of log-stable distributions in the least square sense is enhanced by their better ability to model the tail or small-drops part of the distribution and to fit moments of the distribution. This may actually prove to be important for some modeling of polydisperse multiphase systems built on moments of the distribution: for instance, on the spray intensity, i.e. the volume number of drops, or on the surface volumetric density.

## II. FRAGMENTATION EQUATION AS A MASTER EQUATION FOR THE VOLUME DISTRIBUTION

Experimental results in Sec. III indicate that, totally skewed to the left log-stable volume pdf, are good candidates to describe spray pdf. In this section, we present a possible theoretical justification of this fact. Part of our work is based on Ref. [8], in which a stochastic modeling asymptotically leading to a log-normal number distribution is obtained. We present here a simpler derivation of the equation and generalize these results further, first to volume distributions and second to log-stable pdf.

### A. Derivation of the master equation of the volume distribution

Let $\nu(r)$ be the frequency of disintegration of drops of radius $r$. Let $V(r)$ be the volume occupied by drops of radius $r$ and let, in this fragmentation or atomization process, $q(a).da$ be the fraction of the volume $V(r)$ that creates droplets whose sizes are in the range $r[a,a+da]$. Then the volume lost in a unit time by drops in the range $[r_1,r_1+dr_1]$ is equal to $\nu(r_1)V(r_1)dr_1$, and the resulting volume gained by drops in the range $r_1[a,a+da]$ is equal to $\nu(r_1)V(r_1)dr_1q(a)da$. To obtain the volume of drops in the range $[r,r+dr]$, let us notice that the relation $r_1=r/a$ is a simple change of variable, so that the differential element $dadr_1$ can be written as

$$dadr_1 = \frac{da}{a}dr. \tag{1}$$

Then, assuming that multipliers $a$ belong to the interval [0,1], the volume gained in a unit time by drops in the range $[r,r+dr]$ is equal to

$$\int_0^1 q(a)\nu(r_1)V(r_1)dadr_1 = dr\int_0^1 q(a)\nu\left(\frac{r}{a}\right)V\left(\frac{r}{a}\right)\frac{da}{a}, \tag{2}$$

which leads to

$$\frac{\partial V(r)}{\partial t} = (\mathbf{I}_- - 1)[\nu(r)V(r)], \tag{3}$$

where

$$\mathbf{I}_- F(r) = \int_0^1 F\left(\frac{r}{a}\right)q(a)\frac{da}{a}. \tag{4}$$

It can be first noted that rescaling both sides of Eq. (3) by the total volume of liquid would give the same equation since it is a constant. Therefore up to this rescaling, $V(r)$ can be considered as the volume distribution of diameters $f_v(2r)$:

$$f_v(2r) = \frac{V(r)}{\int_0^\infty V(r)dr}. \tag{5}$$

Second, the hypothesis that $q$ should only depend on the ratio $a$ between the radius of the daughter drops and the parents' drop is a very strong hypothesis called "scaling similarity" in Ref. [14] and "homogeneous kernel" in Ref. [9]. It can be assumed that this is valid for spray with a very high Weber number We and low or finite Ohnesorge number Oh (see Sec. III for a definition of these numbers). Breakup of drops is then mainly governed by their kinetic energy, which can be thought as nearly infinite; $q$ can then be considered as scale independent. This is not the case for lower value of We or very high value of Oh [16]. Finally, let us note that Eq. (3) is very similar to the homogeneous fragmentation equation of Ref. [9] but the kernel is, here, assumed to be continuous and not discrete.

Making, as Kolmogorov [2], the following change of variable:

$$T(x) = V[\exp(x)],\ \tilde{\nu}(x) = \nu[\exp(x)],\ b = \ln(a),\ \text{and}$$
$$\lambda(b)db = q(a)/a\ da$$

yields

$$\frac{\partial T(x)}{\partial t} = (\tilde{\mathbf{I}}_- - 1)[\tilde{\nu}(x)T(x)], \tag{6}$$

$$\tilde{\mathbf{I}}_- \cdot T(x) = \int_{-\infty}^0 T(x-b)\lambda(b)db \tag{7}$$

By making the assumption that

$$\int_{-\infty}^0 |b|^s\lambda(b)db < \infty \quad \text{for} \quad s = 1,2,3 \tag{8}$$

and considering that $\nu$ is constant, Kolmogorov concluded that $T$ is asymptotically normal. However, there is no *a priori* reason to assume (8) and we will drop this hypothesis, considering that

$$\int_{-\infty}^0 \lambda(b)db = 1 \tag{9}$$

and that there exists $1<a\leqslant 2$ such that

$$\int_{-\infty}^0 |b|^s\lambda(b)db < \infty \quad \text{for} \quad s < \alpha, \tag{10}$$

$$\int_{-\infty}^0 |b|^s\lambda(b)db = +\infty \quad \text{for} \quad s \geqslant \alpha. \tag{11}$$

### B. Asymptotic self-similar log-stable solution

Let us recall, see Ref. [17], that Lévy stable distributions are defined from their Fourier characteristic function as follows.

*A random variable X is said to have a stable distribution denoted* $L_\alpha(x;\beta,\sigma,\mu)$ *if there are real parameters* $0<\alpha\leqslant 2$, $0<\sigma$, $-1\leqslant\beta\leqslant 1$, *and* $\mu$ *such that its characteristic function has the following form*:

$$\hat{p}_\alpha(k;\beta,\sigma,\delta) = \exp(ik\mu - \sigma^\alpha|k|^\alpha[1 + i[\text{sgn}(k)]\beta\omega(|k|,\alpha)]), \tag{12}$$

*where*

$$\omega(|k|,\alpha) = \begin{cases} \tan(\alpha\pi/2) & \text{if } \alpha \neq 1 \\ -(2/\pi)\ln|\text{k}| & \text{if } \alpha = 1, \end{cases}$$

and $\alpha$ is the stability index governing the decrease of the tail of the probability, $\sigma$ is the scale parameter analogous to the standard deviation of the normal law, and $\mu$ is the shift parameter governing, but not to be confounded with, the mean of the distribution. $\beta$ is a skewness parameter, $\beta=0$ indicates a symmetric distribution, $\beta<0$ a distribution skewed to the left, and $\beta>0$ a distribution skewed to the right. One can notice that if $\alpha=2$, the distribution is Gaussian and the parameter $\beta$ is meaningless since $\omega(|k|,\alpha)=0$ (normal distributions are not skewed). When $\beta=-1$ totally skewed Lévy distributions are also characterized by their two-sided Laplace transform which reads [17] (for $\alpha\neq -1$ and $q>0$)

$$\langle \exp(-qX)\rangle = \exp\left[ q\delta - \frac{\sigma^\alpha}{\cos\left(\frac{\pi\alpha}{2}\right)} q^\alpha \right]. \tag{13}$$

In order to show that totally skewed stable distributions are suitable solutions of Eq. (6), let us rewrite Eq. (7) as

$$\begin{aligned}\tilde{\mathbf{I}}_- \cdot T(x) &= \int_x^{+\infty} T(b)\lambda(x-b)db \\ &= \int_{-\infty}^{+\infty} T(b)H(b-x)\lambda(x-b)db,\end{aligned} \tag{14}$$

where $H$ is the Heaviside function. Then using two-sided Laplace transform, one gets, taking the constant value $\nu=1$ for the rate:

$$\frac{\partial T(s,t)}{\partial t} = [\lambda(s)-1]T(s,t), \tag{15}$$

where

$$\lambda(s) = \int_0^\infty \lambda(-x)\exp(sx)dx$$

is actually a one-sided Laplace transform due to the introduction of the Heaviside function. This leads to

$$T(s,t) = T(s,0)\exp[(\lambda(s)-1)t]. \tag{16}$$

From hypotheses (9)–(11) and a Tauberian theorem [18], we obtain that the one-sided Laplace transform of $\lambda(-x)$ reads near 0

$$\lambda(s) = 1 + \overline{\lambda}s + \gamma s^\alpha + o(|s|^\alpha), \tag{17}$$

where $\overline{\lambda}$ is the mean of $\lambda(-x)$ and $\gamma$ is a real number which can be written as

$$\gamma = -\frac{\sigma^\alpha}{\cos\left(\frac{\pi\alpha}{2}\right)}.$$

If initially $T(x,0)=\delta_{x_0}$, then $T$ is asymptotically a stable pdf totally skewed to the left. Indeed, in the limit $t\to\infty$, the real part of the term in the exponential is diverging toward $-\infty$ and inverting the Laplace transform, the contribution $e^{-s\cdot x}T(s,t)ds$ to the volume pdf is negligible. This is not the case if we impose $s^\alpha t$ to stay finite and we get the asymptotic result

$$T(s,t) \sim \exp\left( s(x_0 - \overline{\lambda}t) - \frac{\sigma^\alpha s^\alpha t}{\cos\left(\frac{\pi\alpha}{2}\right)} \right) \quad (t\to\infty, s^\alpha t \quad \text{finite}), \tag{18}$$

which is the Laplace characteristic function of an $\alpha$-stable distribution with $\beta=-1$ and scale parameter $\sigma t^{1/\alpha}$. The volume distribution can be interpreted as a pdf for the random variable $x=\ln(r)$. Using scaling properties of Lévy distribution [17], $x/t^{1/\alpha}$ is a stable law of stability index $\alpha$, skewness parameter $\beta=-1$ and scale parameter $\sigma$. So, we can point out that

$$T(x,t) \sim \frac{1}{t^{1/\alpha}} p_\alpha\left(\frac{x}{t^{1/\alpha}}; -1, \sigma, x_0 - \overline{\lambda t}\right) \quad (t\to\infty) \tag{19}$$

is an asymptotic self-similar solution to Eq. (6) using hypotheses (9)–(11). Setting $\alpha=2$ leads to Kolmogorov's previous result.

### C. Distribution of conservative or nonconservative quantity

An equation similar to Eq. (3) has been obtained by Gorokhovski and Saveliev in Ref. [8] for the number $F(r)$ of droplets of radius $r$. But the frequency $\nu$ had to be replaced by $q_0\nu$, where $q_0>1$ is the mean number of droplets resulting from the breakup of the parent drop. Since the total number of droplets $n$ is increasing with time, a normalized number distribution of droplets is defined as $f=F/n$. Taking the moment of Eq. (3) and integrating, they could obtain the following evolution equation for the number of drops:

$$\frac{\partial n}{\partial t} = \nu_0(q_0-1)n,$$

so that the rescaled distribution $f$ satisfied Eq. (3) exactly. We could have chosen the same way and would have obtained that the number distribution may asymptotically be log-stable. However, there is a strong assumption in doing so; the mean number of droplets $q_0$ is meaningful, i.e., finite and independent of the scale $r$ or the ratio $a$. Reasoning directly with conserved quantities, such as volume for instance, circumvents this problem as the rescaling is made using a constant value and the extra parameter $q_0$ is no longer needed.

## III. LOG-STABLE PDF IN A HORIZONTAL ANNULAR GAS-LIQUID FLOW

We are now in position to show that solution (19) can fit experimental pdf. Actually conditions (9)–(11) with $1<\alpha\leqslant 2$ have been inspired by the following experimental results since we could also have chosen $\alpha$ to be in the range $0<\alpha\leqslant 1$. This choice was thus not arbitrary.

### A. Experimental setup

We considered pdf measured by Simmons and Hanratty [12] in a horizontal annular gas-liquid flow. Though the experimental setup is well detailed in Ref. [12] and in references therein, its principle is briefly recalled. The flow loop is an acrylic cylinder 27 m long and of diameter $D_w=0.0953$ m wide. Air and water mass flows and velocities are metered at the entrance and measurements are made 21 m away from the entrance so that the flow can be considered as fully developed. Drop diameters $D$ are measured with a Malvern Spraytec RTS 5008 analyzer. Data were processed on billions of drops giving very smooth pdf over three decades of drop diameters (ranging from less than 1 $\mu$m to 1000 $\mu$m). Important parameters of the experiments are $V_{SL}$, the velocity of the liquid relatively to the solid cylinder, ranging from 2.2 to 13.5 cm/s, and $V_{SG}$, the velocity of the gas relatively to the solid, ranging from 30 to 50 m/s. From these external parameters, one can calculate two important dimensionless numbers [19]:

the global Weber number

$$\mathrm{We} = \frac{\rho_L V_{SG}^2 D_w}{\gamma} \approx 3 \times 10^6,$$

and the global Ohnesorge number

$$\mathrm{Oh} = \frac{\mu}{\sqrt{\gamma \rho_L D_w}} \approx 3 \times 10^{-6},$$

where $\gamma$ is the surface tension of the liquid, $\rho_L$ its density, and $\mu$ its dynamic viscosity.

One of the objectives of this experiment was to show evidence of the stratification of the drop size. A global Froude number, defined as

$$\mathrm{Fr} = \frac{V_{SG}^2}{g D_w} \approx 10^3$$

may characterize this sedimentation effect. Unfortunately, Fr is not, in this experiment, an independent new parameter since the value of the Froude number is proportional to the value of the Weber number. Since a higher gas speed lessens the effect of the gravity, the high value of this nondimensional number indicates that gravitation and sedimentation are not, in this experiment, governing the physical process of atomization. However, sedimentation does actually occur all along the pipe, as reported in Ref. [12]. Therefore, it has been chosen to focus on the 26 measurements made on the centerline where this stratification is less important.

### B. Fitting of the drop spray pdf

We have used original data set provided by Dr. Simmons and Professor Hanratty. These data are given by normalized volume distribution functions $f_v$ which are defined by

$$\frac{dV}{dD} = f_V(D), \tag{20}$$

$$\int_0^\infty f_V(D)dD = 1, \tag{21}$$

$$f_V(D)dD = f_n(D)D^3 dD, \tag{22}$$

where $f_n$ is a non-normalized number distribution.

Traditional empirical volume distributions mentioned above are the log-normal pdf which possesses two free parameters $D_m$ and $\sigma$,

$$f_V(D) = \frac{1}{\sqrt{2\pi} D\sigma} \exp\left( -\frac{1}{2\sigma^2}[\ln(D) - \ln(D_m)]^2 \right), \tag{23}$$

and the log-Weibull pdf with three free parameters $q$, $D_m$, and $X$,

$$f_V(D) = q\frac{(\ln D)^{q-1}}{D(\ln X)^q} \exp\left[ -\left( \frac{\ln D - \ln D_m}{\ln X} \right)^q \right], \tag{24}$$

the modified Rosin-Rammler pdf is a log-Weibull distribution where $D_m = 1$, the upper-limit log-normal (ULLN) with three free parameters $D_{max}$, $a$, and $\delta$,

$$f_V(D) = \frac{\delta D_{\max}}{\sqrt{\pi} D(D_{\max} - D)} \exp\left\{ -\left( \delta \ln \left[ \frac{aD}{D_{\max} - D} \right] \right)^2 \right\}. \tag{25}$$

According to Simmons and Hanratty "*The drops size distributions are best represented by an upper-limit log-normal distributions functions, but there is consistent underprediction of the number of small drops*." Lévy stable pdf may lead to better prediction since they have well-known heavy-tailed distributions.

#### *1. Least square fitting of the pdf computed by FFT*

The pdf is calculated by inversing the Fourier transform (12) through the use of a specific fast Fourierr transform (FFT) algorithm described in Ref.[20]. All four parameters $\alpha$, $\beta$, $\sigma$, $\delta$ were free in order to give the best fit to the experimental data. Except in the fitting of the ULLN where the volume drop pdf is directly employed, the logarithmic experimental pdf is calculated from the bin distribution of the volume; the cumulated volume of drops in the bin $[\ln(d_i), \ln(d_i + 1)]$ is taken to be the probability $p_i$ that the logarithm of the drop diameter is located within this interval. The corresponding density $y_i$ is then $p_i$ divided by the width of the interval. This density is then affected to the midst of the interval. Pdf's are then fitted by minimizing the error function

$$\mathrm{Err}_2 = \sqrt{\frac{\sum (\tilde{y}_i - y_i)^2}{\sum y_i^2}}, \tag{26}$$

where $y_i$ are the measurements and $\tilde{y}_i$ the results of the fitting. The algorithm used is the implementation of the Levenberg-Marquardt algorithm delivered in Matlab™. Figure 1 shows the result of such a fitting. Comparison is made between log-stable, upper-limit Evans, log-Weibull, and log-normal pdf. On the lower diagram the pdf has been represented on a logarithmic scale putting an emphasis on the tail of the distribution. It is clear that log-stable laws cope better with the left tail of these pdf's.

The relative error $\mathrm{Err}_2$ corresponding to the different fittings is represented in Fig. 2. Experiments 1–8 are made with $V_{SG}$ equal to 30 m/s, experiments 9–14 with $V_{SG}$ equal to 35 m/s, 15–20 with 43 m/s, and 21–26 with 50 m/s. The performance of the log-stable law is roughly equivalent to the log-normal law for the lower value of the air velocity and the value $\alpha = 2$ has actually been found several time (experiment numbers 1, 2, 6, and 7). For higher velocity, the better overall performance of the log-stable pdf is clear. However, this law possesses *a priori* four independent parameters and should obviously perform better than laws, like upper-limit Evans and log-Weibull, possessing only three parameters or like log-normal law possessing two parameters. Neverthe-

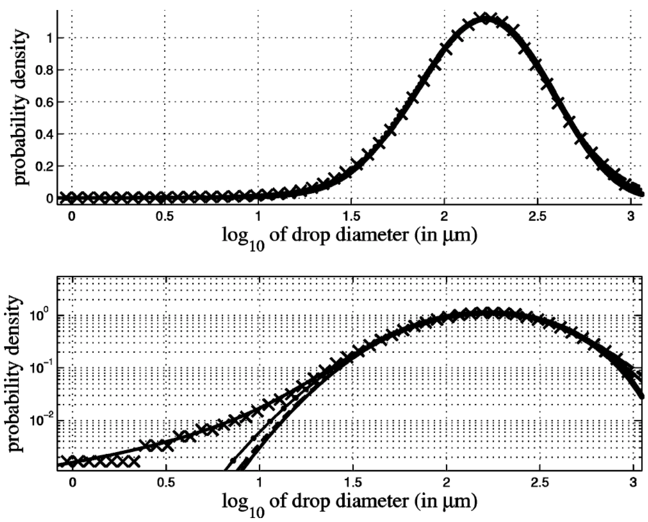


FIG. 1. Fitting of the drop spray pdf; $V_{SL}=0.041$ m/s and $V_{SG}=35$ m/s. From left to right the pdf's are log-stable pdf, upper-limit Evans pdf, log-normal pdf, and log-Weibull pdf.

less, except for the experiment number 21 where $\alpha=1.83$ and $\beta=-0.80$, and number 22 where $\alpha=1.81$ and $\beta=-0.94$, the same value $\beta=-1$ has been found. This agrees with our modeling and it should be no longer considered as an independent parameter. (Another exception was when the value $\alpha=2$ has been obtained but it has already been seen that $\beta$ is meaningless in this case.) It is interesting to note that totally skewed to the left ($\beta=-1$) log-stable distributions are the only log-stable distributions that possess finite moments of all positive orders.

### *2. Moments of the volume distribution*

The ability of log-stable distributions to tackle with the small drops side of the distribution can be demonstrated in another way by their better restitution of moments and especially negative moments of the distribution. Theoretically, moments of the volume distribution are defined by Eq. (27).

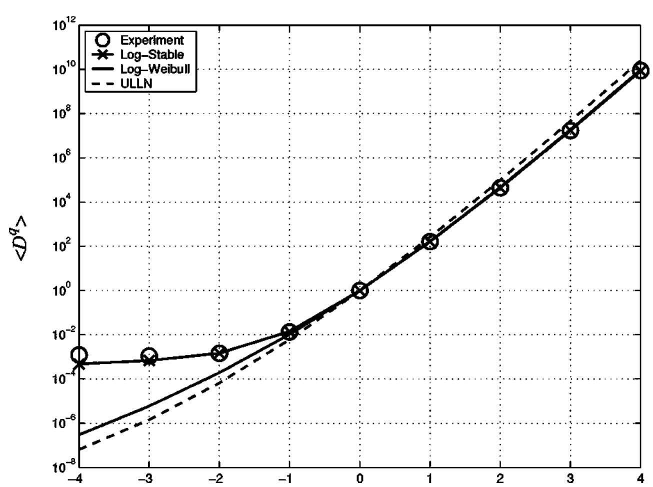


FIG. 2. Comparison of the relative error for the different fitted pdf's. Note that the velocity of the gas is increasing with the index number of the experiment.

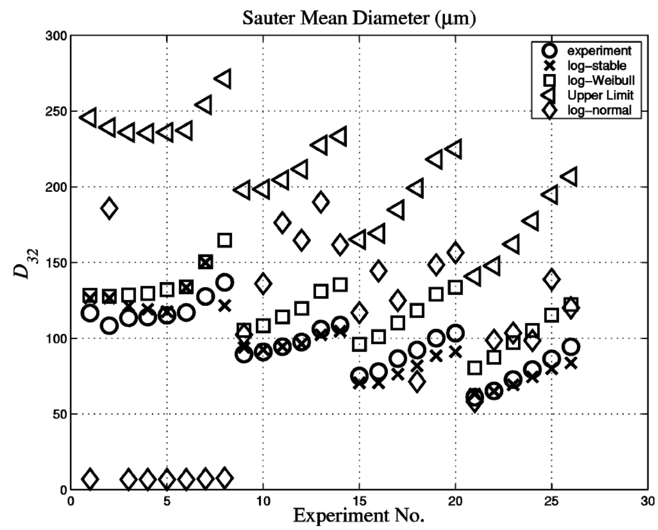


FIG. 3. Moment $\langle D^q\rangle_V$ of the drop spray pdf. Comparison between experimental and several fitted pdf's; $V_{SL}=0.022$ m/s and $V_{SG}=43$ m/s.

$$\langle D^q\rangle_V^{exact}=\int_0^\infty D^q f_V(D)dD. \tag{27}$$

One can note that, due to the power-law tail of skewed stable distribution for large negative values, exact negative moments are rigorously infinite. Actually, this singularity is smoothed out by surface tension or viscosity, which prevents the appearance of drops under a given size (related to We $\sim 10$ [19] or $Re\sim 1$). So, our "asymptotic solution" should rather be considered as an "intermediate asymptotic solution."

Moreover, in order to prevent from taking into account large or small drops that have not been measured in the experiment, the range needs to be restricted to the measurement range $[D_{min},D_{max}]$. Thus moments plotted in Fig. 3 have been calculated using experimental *volume pdf* and fitted *volume pdf* through relation (28):

$$\langle D^q\rangle_V=\int_{D_{\min}}^{D_{\max}} D^q f_V(D)dD. \tag{28}$$

Small drops have a greater importance than large drops for negative moments of the distribution whereas large drops dominate positive moments. Therefore, the poor performance (cf. Fig. 3) of empirical pdf for negative moments can be related to their poor modeling of the small drop part of the pdf. For positive moments and for large values of the moment order $q$, $1_{[D_{\min},D_{\max}]}(D)D^q f_V(D)$ is quickly behaving like $D^q\delta_{D_{\max}}(D)$. This means that the "truncation" effect is very important and that positive moments are quickly behaving as $D_{max}^q$. As a result, curves of Fig. 3 are asymptotically straight lines whereas they should be power laws (as for exact positive moments of log-stable distributions).

### *3. Sauter mean diameter*

Practically, the preceding moment fitting is of special interest in moment-based spray modeling. One of the most

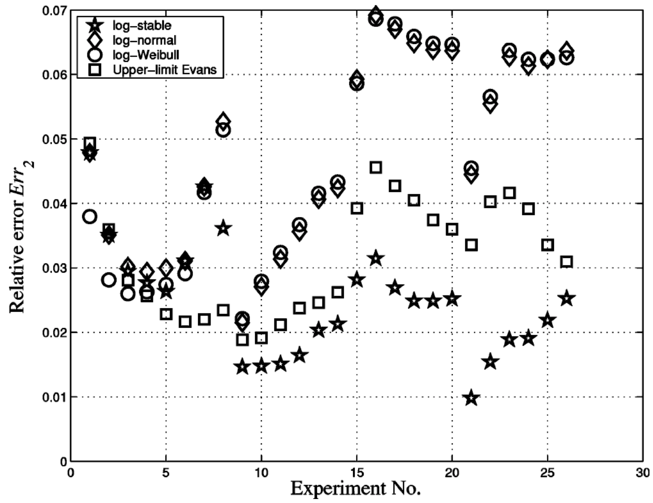


FIG. 4. Sauter mean diameter (in $\mu$m). Comparison between experimental results and values given by several fitted pdf's.

useful moments is the Sauter mean diameter [1], often written as $d_{32}$. It is the diameter of the drop having the same area to volume ratio as the whole liquid. It can be expressed using the *number distribution* or equivalently the *volume distribution* using relation (29):

$$d_{32} = \frac{\int D^3 f_n(D)dD}{\int D^2 f_n(D)dD} = \frac{\int f_V(D)dD}{\int D^{-1} f_V(D)dD} = \frac{1}{\int D^{-1} f_V(D)dD}. \tag{29}$$

Written with the *volume distribution*, it involves a negative moment. Since truncated log-stable distributions better fit moments and especially negative moments of the volume distribution than standard distributions, they are more efficient in calculating the Sauter mean diameter of the spray [15]. This can be verified in Fig. 4.

#### *4. Variation of the parameters with We and $V_{SL}$*

Figures 5 and 6 show the evolution of the stability index $\alpha$ and scale parameter $\sigma$ versus two external parameters: We and $V_{SL}$. The fitted values of $\alpha$ are located in the interval ]1.75, 2]. Actually, we can notice that when the gas flow rate is increased, $\alpha$ decreases and the distribution is getting more and more heavy tailed since totally skewed to the left stable pdf are decreasing like $1/|x|^{(\alpha+1)}$ for $x \to -\infty$. This can be related to Simmons and Hanratty's following remark [12]: "*There is a clear increase in the importance of small drops as the gas flow rate is increased*."

These fittings do not exhibit any clear similarity law relating $\alpha$ and $\sigma$ to We and $V_{SL}$. In homogeneous isotropic turbulence, log-stable models lead to a constant value of $\alpha$ equal to 1.65 and $\sigma^\alpha$ can be related to $\ln(Re)$ or $\ln(Re\,\lambda)$ [9,10,21]. But comparatively, we cannot make here any homogeneity hypothesis, since larger drops and smaller drops are eventually separated by sedimentation (though this effect is not dominant). This sedimentation effect varies with the

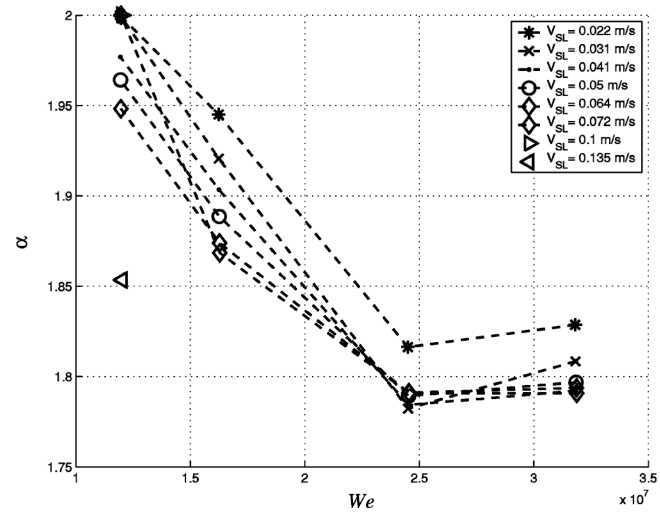


FIG. 5. Stability index $\alpha$ found for different values of We and $V_{SL}$.

Froude number in an unknown way, hidden moreover by its proportionality to the Weber number.

## IV. CONCLUSION

We have shown that log-stable distributions are able to describe drops spray pdf in an efficient way. They actually perform at least as well as pdf classically used in atomization and seem to be much better for high Weber number sprays. Assuming that they are totally skewed to the left, they possess no more independent parameters than empirically constructed pdf. Moreover, all of them can be assigned a physical interpretation: the stability index $\alpha$ governs the tail of the distribution, the scale parameter $\sigma$ governs (in combination with $\alpha$) the width of the distribution, and the shift parameter $\mu$ (in combination with $\alpha$ and $\sigma$) governs the mean of the distribution.

The small sedimentation of drops in the experiment prevents any reasonable correlations between parameters of the

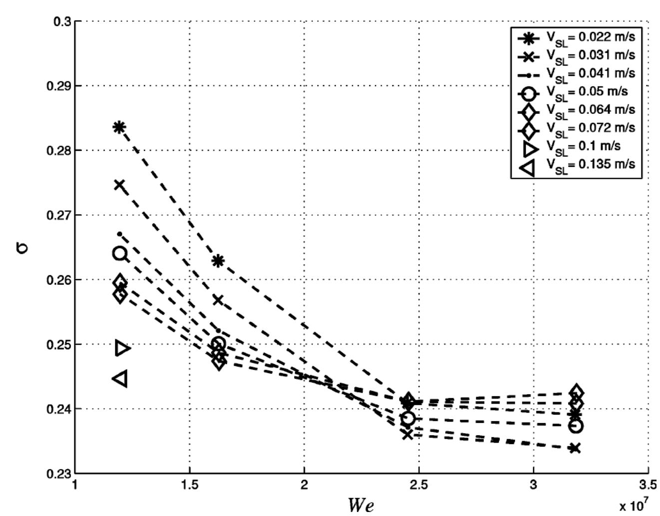


FIG. 6. Scale parameter $\sigma$ found for different values of We and $V_{SL}$.

distribution and external parameters like gas Weber number and liquid mass flow; even less with a universal nature. One can notice that values of $\alpha$ given in Fig. 3 are not related to the value $\alpha = 1.65$ found by Kida for the dissipation pdf of turbulence. Yet, there are firm convictions that the size of drops in a turbulent spray should be correlated in some way to the size of eddies in the liquid [22]. Nevertheless, direct comparison with turbulence cascade model is difficult as turbulence does actually occur in both gas and liquid and momentum exchange between both phases is complex and important in the mechanism of drop breakup. As previously noticed by Novikov and Dommermuth [6], it seems difficult to build a bridge between cascade in drop breakup and in eddy breakup without making correlated measures of drop radii and turbulence dissipation.

Lastly, it is common knowledge in atomization studies that "*the fluctuations of the total number of droplets about the mean [are not] negligible*" [23]. Having a look at the distributions, normal distributions are rather narrow whereas log-normal are somewhat wider. This makes simulations of log-normal cascade difficult and some authors have tried to replace them by the close-looking but narrower Gamma distribution [24]. Lévy stable distributions are broader and the drop size volume pdf (and not the volume pdf of the logarithm of the drop size that we actually studied) is eventually diverging at zero [10], making the volume density of drops of arbitrarily small size, arbitrarily large. This is not a shortcoming since the probability of any bin remains finite (the pdf is integrable). Moreover, the cascade mechanism and the drop diameter distribution should be bounded from below by some kind of minimum scale given by either $\mathrm{We} \sim 10$ [19] or $Re \sim 1$, whichever leads to the largest scale. This may vary from liquid to liquid, since viscosity may, in some case, prevent the formation of drops, before surface tension (for high Ohnesorge number).

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